

Lec 6:

09/08/2014

## Particle Acceleration (Cont'd):

### Fermi Acceleration:

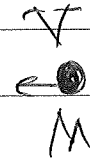
Gas flowing into a compact object is often supersonic. This means that sound waves form discontinuous jumps across the resultant shock front. The diffusion of particles across the shock accelerates them and produces a power-law distribution, which is called the "Fermi acceleration". The accelerating particles scatter off the random fluctuations in the magnetic field (which is a collisionless process). This mechanism constitutes the second electromagnetic acceleration scheme.

To see how random scattering of particles between molecular clouds in the interstellar medium can accelerate them, let's consider a simple one-dimensional version of the problem.

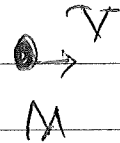
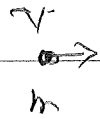
(2)

We consider a large number of massive particles with mass  $M$  (the clouds) moving with the same speed  $V$  but randomly (i.e., back and forth). They collide with a small test particle of mass  $m$ , whose velocity is  $v$ . There are two types of collisions in this set up:

(1) Head-on Collision (h):



(2) Catch-up collision (c):



Here we assume that  $m \ll M$  and  $v \gg V$ .

By using the conservation of momentum and energy, one can show that the energy of the test particle changes by respective amounts  $\Delta E_h$  and  $\Delta E_c$  in head-on and catch-up collisions, where:

$$\Delta E_h = \frac{1}{2} m (v + 2V)^2 - \frac{1}{2} m v^2 = 2m v V + 2m V^2$$

$$\Delta E_c = \frac{1}{2} m (v - 2V)^2 - \frac{1}{2} m v^2 = -2m v V + 2m V^2$$

(3)

It is seen that the test particle gains energy in head-on collisions and loses energy in catch-up collisions. To find the net increase in the energy, we should also include the probability for each type<sup>of</sup> collision. The probability is proportional to the collision rate, which is in turn proportional to the relative velocity of colliding particles, we have:

$$P_h = \frac{v_+ V}{2V}, \quad P_c = \frac{v_- V}{2V}$$

Thus, the average increase in energy per collision is:

$$\langle \Delta E \rangle = P_h \Delta E_h + P_c \Delta E_c \Rightarrow \frac{\Delta E}{E} = 4 \left( \frac{V}{v} \right)^2$$

This is a second-order effect, which is too small to yield a significant energy increase. However, as we will see, a shock geometry improves the situation by producing first-order dependence of  $\Delta E$  on the velocities.

• Before turning to this, we can also find the energy spectrum,

(4)

resulting from this stochastic process by using the one-dimensional

model. The energy distribution of particles satisfies the

diffusion-loss equation:

$$\frac{dN(E, \eta, t)}{dt} = - \frac{\partial \Phi_s}{\partial \eta} - \frac{\partial \Phi_E}{\partial E} + \frac{\partial N}{\partial t}$$

Here  $N(E, \eta, t)$  is the distribution function, and  $N dE d\eta$  is the number of particles between  $\eta$  and  $\eta + d\eta$  whose energy is

between  $E$  and  $E + dE$ .  $\Phi_s$  is the spatial flux, which can

be written in terms of a diffusion coefficient  $D$ :

$$\Phi_s = - D \frac{\partial N}{\partial \eta}$$

$\Phi_E$  is the energy flux, and is given by:

$$\Phi_E = N \frac{dE}{dt}$$

Here  $\frac{dE}{dt}$  is the rate of energy increase due to acceleration

of particles. It follows,

$$\frac{dE}{dt} = \langle \Delta E \rangle \nu = 4\nu \left( \frac{V}{v} \right)^2 E \quad (\nu: \text{frequency of collisions})$$

(5)

The term  $\frac{\partial N}{\partial t}$  represents the explicit time variation in  $N$ .

We can simplify the diffusion-loss equation by ignoring diffusion and setting  $\frac{\partial N}{\partial t} = -\frac{N}{\tau}$ , where  $\tau$  is the characteristic time scale that particles spend in the acceleration zone. We

then find:

$$\frac{dN}{dt} \approx -\frac{\partial}{\partial E} (N \alpha E) - \frac{N}{\tau} \quad \alpha \equiv 4\pi \left(\frac{v}{c}\right)^2$$

In equilibrium, we have  $\frac{dN}{dt} = 0$ , which gives rise to:

$$\frac{\partial (N \alpha E)}{\partial E} \approx -\frac{N}{\tau} \Rightarrow \alpha N + \alpha E \frac{\partial N}{\partial E} \approx -\frac{N}{\tau} \Rightarrow \frac{\partial N}{\partial E} \approx -\left(1 + \frac{1}{\alpha \tau}\right) \frac{N}{E}$$

If  $\alpha \tau$  is constant, we find:

$$N(E) \approx N_0 E^{-\left(1 + \frac{1}{\alpha \tau}\right)}$$

This is a remarkable result that demonstrates the energy spectrum is a power-law function, as seen in the case of cosmic rays and many high-energy sources. The spectral index

$-\frac{1}{1 + \alpha \tau}$  may vary slightly from location to location.